Computational Assessment of Electrostatic Field Profiles Near High-Voltage Transmission Systems in Malaysia via Finite Difference and Crank-Nicolson Schemes

Salaudeen Abdulwaheed Adebayo[a], Saratha Sathasivam[a,*], Majid Khan Bin Majahar Ali[a], Ummi Umara Binti Ahmad Fuad[a] , Hasliza Binti Abbas[a] and Muraly Velavan[b]
[a] *School of Mathematical Sciences, Universiti Sains Malaysia, Penang, 11800, USM, Malaysia.*
[b] *General and Foundation Studies, AIMST University 08100 Bedong Kedah, Malaysia*

Abstract

Unconstrained exposure of humans and their immediate environments to electrostatic fields generated by high-voltage transmission lines has raised a lot of concerns regarding public health safety. These transmission lines, often sited near human residential and urban areas, may pose long-term health risks depending on the strength and duration of exposure. Various studies have linked prolonged exposure to electromagnetic fields to various health conditions, including neuropsychological disorders, cardiovascular diseases, and central nervous system complications. While high-voltage transmission lines are essential for efficient power distribution, their proximity to populated areas necessitates regulatory policies to mitigate potential risks. This study aims to analyse the spatial variation and intensity of the electrostatic field distribution around high-voltage power transmission lines in Malaysia, using two numerical methods, considering the country's infrastructure features and regulatory emphasis on public exposure limits. The Finite Difference Method (FDM) and the Crank-Nicolson Method (CNM) are applied to solve Laplace's Equation, which governs electrostatic potential, field intensity and distribution. Factors such as voltage levels, tower configurations, and conductor height are considered in the analysis. The study compares the accuracy, convergence rate, computational efficiency, and execution time of both numerical techniques to determine which of the methods is more suitable to solve such a problem. Our result demonstrates that FDM is fundamentally more suited for solving the Laplace equation governing electrostatic potential, field intensity, and spatial distribution due to its direct discretisation of spatial derivatives while using CNM in this context only introduces unnecessary complexity and computational overhead without providing any benefits in returns. The study provides insights into safe management practices by identifying critical zones of elevated electrostatic field intensity, indicating minimum safe distances for human exposure, and supporting infrastructure planning in accordance with Malaysian regulatory standards.

Keywords: *Electrostatic field, High-voltage transmission lines, Electromagnetic exposure, Numerical analysis, Health risks*

* Corresponding author.
E-mail address: salaudeenadebayo@gmail.com

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1. Introduction

Continuous exposure of humans and their immediate physical environment to both electromagnetic fields and electrostatic fields generated by high-voltage transmission lines has raised significant concerns regarding human well-being, as these high-voltage transmission lines are often situated close to residential and urban areas, a decision that might pose a long-term effect on residents of such areas [1]. The impact of this exposure on human health may vary depending on the strength of the electromagnetic fields these systems produce, the length of exposure to the radiation, and the distance from these electrical systems [2],[3]. Conceptually, it is known that stronger electric fields produced by a higher voltage transmission line can pose a danger to people's health, animals' health, and the environment. Even though it is asserted that it decreases as one moves away from the source of radiation, this can be linked to the consequence of Coulomb's law [4],[5]. The sensitivity of the issue regarding human lives has made governments of many nations formulate necessary programs and policies aimed at controlling the damaging effects of such exposure and investigating the extent of the detrimental effects in residential areas where exposure to electromagnetic fields is more pronounced [6],[7]. Under such guidelines, every nation is free to propose its own electromagnetic exposure rules and laws, guided by standards established by groups like the International Commission on Non-Ionizing Radiation Protection (ICNIRP). According to ICNIRP, people should not be subjected to electric fields that are stronger than 5 kV/m at a frequency of 50 Hz [8].

Electric fields are majorly generated by the electric currents flowing through cables, conductors and equipment. Employees in most power plants believe that the regulations are vague, organisational rules are not explicitly stated, there is a lack of clarity on the properties of the electromagnetic field (EMF), and there is a need for potential health effects and mitigation methods of EMFs [9], [10]. Various studies have investigated the impact of induced currents in the human body caused by 50 Hz electric fields from high-voltage transmission lines [11], [12]. Most research highlighted a concern about the risk to human health and the biological effects that such electric fields of 50 Hz can induce in humans. [2] reported that stronger electromagnetic fields produced by higher-voltage carry conductors can undoubtedly harm people, animals, and the physical environment. [13] asserted that exposure to electromagnetic fields from high voltage lines has been linked to various health issues ranging from neuropsychological disorders, anaemia, hoarseness, blood fat increment, fatigue, and depression, to anxiety, potentially leading to more severe long-term health problems like cardiovascular and central nervous systems, with risks increasing depending on the strength and duration of exposure [14].

However, it should be noted that transmission lines, which are vital components of electrical power systems are utilised to transmit electrical power from electrical generation plants to sub-power stations or directly to the final consumers, majorly industries and households. High-voltage cable systems built to carry bulky quantities of electricity over long distances with negligible loss are not positioned within the residential area intentionally to cause harm or threaten human health, but they are sited to meet the energy requirements of the people and industries, ultimately to reduce operational costs and minimise land and resource usage while maintaining high standard planning constraints among other beneficial factors. Urban and industrial areas need large amounts of electricity, making closeness to transmission lines, power stations, substations, and distribution networks strategically positioned near population centres to minimise power losses during transmission and facilitate efficient power delivery services. Inappropriate positioning of electrical stations can be controlled by making policies and programs that can reduce the damages and promote proper management of health-related issues caused by high-voltage power transmission lines by adhering to the standard approaches and procedures, which include policy-making, social capacity building, and technical solutions among others [15], [16].

This research aims at deploying two prominent numerical methods, namely, the Finite Difference Method (FDM) and the Crank-Nicolson Method (CNM), to investigate the electrostatic field distribution around high-voltage power transmission lines in Malaysia. Electric fields near overhead power lines can be influenced by several factors, including the quantity of the voltage in the line, the configuration of the towers, and the height of the conductors above the ground, among others. These factors are crucial in determining the intensity and distribution of electric fields, which have implications for safety and compliance with regulatory standards, even though it has been reported that the influence of electromagnetic fields can be reduced by various objects such as vegetation and buildings [17]. Accurately modelling the electrostatic field distribution around high-voltage power transmission lines is significant in ensuring the reliability and safety of Malaysia's power grid, infrastructure and the populace. The governing equation for electrostatic potential is Laplace's Equation. Laplace's Equation is the partial differential equation (PDE) that requires numerical solutions due to the complexity of real-world geometries and boundary conditions. Several numerical methods have been proposed, among which are FDM and CNM. Although CNM is traditionally more suited for solving time-dependent PDEs such as the heat and diffusion equations and many more, it is an implicit method that is unconditionally stable and has second-order accuracy in both time and space. It can advantageously be applied to iteratively solve steady-state problems. In this study, CNM is adapted as a pseudo-time stepping method to iteratively converge to the steady-state solution of Laplace's Equation, effectively treating the electrostatic field distribution as the asymptotic solution of a diffusion-like process. The solution of these numerical methods will be compared based on the accuracy of electric field distribution, convergence rate, and computational efficiency, as well as the applicability to realistic conditions in Malaysia's power grid infrastructure, including scenarios with and without space charge effects.

2. Laplace's equation

A broad variety of physical processes involving continuous variables are often described using partial differential equations (PDEs), which are basic and common mathematical models [18]. PDEs include functions of several variables and their respective partial derivatives, in contrast to ordinary differential equations (ODEs), which deal with functions of a single independent variable. PDEs are frequently categorised according to their linearity, order, and solution type. The general representation of PDEs is shown in equation (1).

$$F\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \ldots\right) \tag{1}$$

where $u(x, y)$ is the unknown function to be computed, and its partial derivative describes how the unknown function u with respect to independent variables x and y. [19] asserted that PDEs are broadly classified into elliptic, parabolic and hyperbolic PDEs, among which Laplace's equation is an exemplar elliptic PDE that controls a variety of steady-state physical systems, especially those involving heat conduction, fluid flow, and electrostatics, among others.

Laplace's equation is a popular second-order PDE that is often written as shown in equation (2):

$$\nabla^2 \phi = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{2}$$

where ∇^2 denotes the Laplacian operator and $\phi(x, y, z)$ is the potential unknown function. This equation has been deployed to model the equilibrium state of phenomena characterised by no sinks or sources used in engineering and physics. They are often used together with boundary value problems (BVPs), and the solution to such problems is calculated based on conditions provided along the domain boundaries. The uniqueness of the solutions to such problems in physical science, physics, and engineering is often attributed to the usage of special forms of boundary conditions like Dirichlet's, Neumann's, or mixed boundary conditions [20].

Laplace's equation can be used to describe the electric potential $V(x, y, z)$ in the region where there are no free charges, while is related to the Gauss law. The connection between charge density (ρ) and electrical potential is presented in the Poisson equation shown in equation (3):

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} \tag{3}$$

Setting $\rho = 0$ reduces the equation to Laplace equation (4):

$$\nabla^2\phi = 0 \tag{4}$$

Equation (4) is often deployed in designing capacitor plates, calculating electric field distributions, and solving problems related to electrostatic shielding. The electrical field distribution using Laplace equations is explored in this research since high-voltage transmission lines are critical components of Malaysia's power infrastructure. Ensuring the accurate computation of electric field distribution around these lines is essential for improving insulation design, minimising corona discharge losses, and maintaining system reliability. With the increasing demand for electricity and the expansion of the grid, this problem is particularly relevant for ensuring the sustainability and efficiency of power transmission in Malaysia.

The Laplace equation in two dimensions (2D) is a fundamental partial differential equation (PDE) that is considered in the study. It takes the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{5}$$

With the boundary's conditions:

- Transmission line conductors are modelled as fixed-potential regions (e.g., ϕ=+137.5 kV, −137.5 kV).
- The ground is modelled as ϕ=0.
- Far-field boundaries are assumed to be ϕ=0 to approximate infinity.

3. Electric field distribution analysis using numerical method

Determining the electric field distribution involves solving Laplace's equation with unique boundary conditions that describe the boundary [21]. Solving simple geometric structures such as spheres, cylinders, or parallel plates is straightforward due to the easy derivation of their mathematical formulas. In the real world, high-voltage applications revolve around complex structures, that have irregular shapes, multiple conductors, sharp edges, and varying material properties, making the mathematical equations extremely difficult or impossible to solve directly. Numerical techniques offer an efficient method of analysing and computing the electric field distribution, voltage distribution, and charge accumulation for these intricate geometries. The presence of multiple independent variables and their partial derivatives in PDEs makes them significantly more complex when compared to ODEs. Usually, complex geometries and boundary conditions make solving PDEs analytically impractical or impossible.

Numerical methods are often seen as a stronghold alternative for approximating solutions to PDEs [22]. Methods such as the Finite Element Method (FEM), Finite Difference Method (FDM), Method of Moments (MoM), and many more fall into this category. Most people use these methods to solve complex problems where analytical solutions are unattainable. Most of these methods commence by discretising the problem domain into manageable rudiments that can be deployed to provide an approximation suitable as the solution to electric field distribution in regions with complex geometries. One of the most popular and widely used numerical techniques is the FDM, which is particularly simple and good for problems with heterogeneous features, materials, or irregular boundaries. FDM divides the core area into a mesh of finite regions. Equipped with straightforwardness and adaptability, this method can be used to analyse electric fields in multi-scale systems, like microelectromechanical systems (MEMS) and high-voltage equipment. It is a flexible method for static electric field analysis and can also handle nonlinear materials. However, the computational cost might be substantial, particularly for large-scale problems that need high-performance computer resources and effective methods. CNM is another important numerical method that is especially suitable for the time-domain characterisation of electromagnetic fields. CNM can be used to simulate the propagation of electric fields over time by discretizing space and time and iteratively solving Laplace equations. When examining fleeting events, like electromagnetic pulses or wave propagation in complicated mediums, this approach works incredibly well since problems are broken down into smaller, more manageable components, and one can utilise computer programs to solve them iteratively, using any iterative technique of solving a system of linear equations. Despite being computationally demanding, CNM offers important insights into the dynamic behaviour of electric fields, which is crucial for applications such as wireless communication, optical devices, and radar systems. In this study, we deployed both the FDM and the CNM to solve the Laplace equation and compare their simplicity, accuracy, and computational efficiency.

4. Finite difference method (FDM) for Solving Laplace's equation

FDM is one of the prominent and popular numerical techniques used to find the solutions to partial differential equations (PDEs); this technique systematically replaces derivatives with equivalent finite difference approximations. It is generally used in HV engineering for analysing electric field distribution, potential distribution, and insulation design in complex geometries where analytical solutions are impractical. In the case of electric field distribution. Laplace's equation in electrostatics (for a charge-free region), from equation (4); we have the representation in cartesian Coordinates as shown in equation (5) through deploying central difference approximations and replacing the second-order derivatives with discrete differences such that

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u(x+h,y) - 2u(x,y) + \mathrm{u}(x-\mathrm{h},\mathrm{y})}{h^2} \tag{6}$$

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{u(x,y+h) - 2u(x,y) + \mathrm{u}(x,y-h)}{h^2} \tag{7}$$

The assumption of a uniform grid across the region validates $\Delta y = \Delta x = h$, the governing equation can be substituted into equation (5) and simplified.

$$u_{i,j} \approx \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1}, j + u_{i,j-1}}{4} \tag{8}$$

where $u_{i,j}$ denotes the function value at the grid point, provided that boundary conditions are known. The boundary conditions, which are normally deployed at the grid points as well as the interior points, are essential for obtaining a useful and physically meaningful solution. The discretisation of equation (8) produces a tridiagonal matrix, where the solutions correspond to the interior grid points. This transformation produces a system of linear equations defined by the grid points within the spatial domain. These equations can be solved using various iterative numerical methods, such as the Jacobi method, Gauss-Seidel method, or Successive Over-Relaxation (SOR), among others.

Equation (8), which is often regarded as the finite difference scheme, is solved iteratively using any of the iterative methods: Jacobi method, Gauss-Seidel method, or SOR, each having its own strength, weakness, and convergence properties. The iterative method begins with an initial guess value for each of the variables for the potential values at the grid points; equation (8) is applied at each interior point to update the potential values. This process is repeated until the potential values converge to a stable solution, meaning the change in potential values between iterations is below a specified tolerance. For the Jacobi method, the new value at each iteration is calculated using the previous iteration and it is said to converge if there is the least difference in the solution of the subsequent values or when the stopping criteria have been met.

$$u_{i,j}^{new} = \frac{1}{4}\left(u_{i+1,j}^{old} + u_{i-1,j}^{old} + u_{i,j+1}^{old} + u_{i,j-1}^{old}\right) \tag{9}$$

Whereas, in the case of the Gauss-Seidel technique, the value of the variable is used immediately it is computed or obtained.

$$u_{i,j}^{new} = \frac{1}{4}\left(u_{i+1,j}^{new} + u_{i-1,j}^{new} + u_{i,j+1}^{new} + u_{i,j-1}^{new}\right) \tag{10}$$

The scheme using equation (10) converges faster; we utilise the Gauss-Seidel method in this research, even though the choice of iterative solver depends on the specific problem and desired convergence properties.

5. Crank-Nicolson method for solving Laplace's equation

The Crank-Nicolson method belongs to a class of implicit finite difference methods applicable for time-dependent PDEs, such as the heat equation. However, Laplace's equation is a steady-state (time-independent) equation, meaning the Crank-Nicolson method does not directly apply to solving Laplace's equation. Instead, it is useful for solving related parabolic equations, like the diffusion equation, which converges with Laplace's equation over a period of time. It is known that the Crank-Nicolson method is a time-centred implicit scheme; it averages forward and backward Euler methods for time-stepping, such that:

$$\begin{aligned}\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} &= \frac{1}{2}\left(\frac{u_{i+1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i-1,j}^{n+1}}{h^2} + \frac{u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}}{h^2}\right) \\ &+ \frac{1}{2}\left(\frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{h^2} + \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{h^2}\right)\end{aligned}$$

Rearranging the terms, we have:

$\left(1+\frac{\Delta t}{h^2}\right)u_{i,j}^{n+1} = u_{i,j}^{n} + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n} + u_{i-1,j}^{n} + u_{i,j+1}^{n} + u_{i,j-1}^{n}\right) + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n+1} + u_{i-1,j}^{n+1} + u_{\mathrm{i},j+1}^{n+1} + u_{i,j-1}^{n+1}\right)$ since $\left(1+\frac{\Delta t}{h^2}\right)$ is a constant numerical value, it can be written such that $\alpha = \left(1+\frac{\Delta t}{h^2}\right)$

$$\alpha\left(u_{i,j}^{n+1}\right) = u_{i,j}^{n} + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n} + u_{i-1,j}^{n} + u_{i,j+1}^{n} + u_{i,j-1}^{n}\right) + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n+1} + u_{i-1,j}^{n+1} + u_{\mathrm{i},j+1}^{n+1} + u_{i,j-1}^{n+1}\right)$$

$$u_{i,j}^{n+1} = \frac{1}{\alpha}\left[u_{i,j}^{n} + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n} + u_{i-1,j}^{n} + u_{i,j+1}^{n} + u_{i,j-1}^{n}\right) + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n+1} + u_{i-1,j}^{n+1} + u_{\mathrm{i},j+1}^{n+1} + u_{i,j-1}^{n+1}\right)\right] \tag{11}$$

Equation (11) represents the recessive scheme for the Crank-Nicolson technique, and it is the required equation to be solved in order to obtain the value of the grids within the area of consideration, using the boundary conditions in equation (12).

$$\begin{cases} left\ boundary\,(\phi = 100) \\ Right\,boundary\,(\phi = -100) \\ Top\,and\,bottom\ boundaries\,(\phi = 0) \end{cases} \tag{12}$$

We deployed the same iterative technique (Gauss-Seidel) to solve the system of the linear equation from the scheme in order to obtain the value at each grid.

6. Procedures for the two numerical methods

Step-by-step procedures for solving the Laplace equation using FDM and CNM are as follows:

Step 1: Initialization of the computational domain into a grid of points (spacing) Δx and Δy, for both techniques and boundary conditions.

Step 2:

(a) Discretization, for the Finite Difference method approximates the Laplace equation (5) using the central difference formula,

$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1,j} - 2u_{ij} + \mathrm{u}_{i-1,j}}{(\Delta x)^2}$ and $\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i,j+1} - 2u_{ij} + \mathrm{u}_{i,j-1}}{(\Delta y)^2}$ substitute into Laplace equation and simplify to have $u_{ij} = \frac{1}{4}\left(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right)$

(b) Discretization, for the Crank-Nicholson scheme we use the central difference for the spatial derivative $\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}}{(\Delta x)^2}$ and $\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{(\Delta y)^2}$, we then use a forward difference for the time derivative $\frac{\partial u}{\partial t} \approx \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t}$ on substituting and simplifying we have

$$u_{i,j}^{n+1} = \frac{1}{\alpha}\left[u_{i,j}^{n} + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n} + u_{i-1,j}^{n} + u_{i,j+1}^{n} + u_{i,j-1}^{n}\right) + \frac{\Delta t}{2h^2}\left(u_{i+1,j}^{n+1} + u_{i-1,j}^{n+1} + u_{\mathrm{i},j+1}^{n+1} + u_{i,j-1}^{n+1}\right)\right]$$

Step 3:
(a) For FDM, formulation of a system of equations and expression of the discretized equations in matrix form for numerical computation of each grid value
(b) For CNM, step (2b) the system of linear equations can be expressed in the form $Au^{n+1} = Bu^{n}$ where A is a matrix containing coefficient for unknowns at a time step $n+1$, B is a matrix containing coefficient for known values (based on boundary condition at a time step n, while u^{n+1} and u^{n} are vectors of unknown and known values respectively.

Step 4: Deployment of boundary conditions, in our case, we apply the Dirichlet boundary condition, and simplify the expressions using a uniform grid ($\Delta x = \Delta y$). This forms a system of linear equations.

Step 5: Solve the system of linear equations using Gauss-Seidel iterative method

Step 6: Set initial guess for the grid points and iterate for convergence, perform the convergence test, and continue iterations until the tolerance value is attained and convergence is achieved

Step 7: Output the result, visualize, table and analyze the results using contour plots and 3D surface plots.

7. Flowchart

Figure 1 and Figure 2 show the flowcharts illustrating the procedures of solving the Laplace equations using the finite difference method and the Crank-Nicolson method.

8. Results and discussion

This study investigates the electrostatic potential distribution around high-voltage power transmission lines in Malaysia using two numerical methods for partial differential equations (PDE), which are the finite difference method (FDM) and the Crank-Nicolson method. This analysis provides insights into the behaviour of electric fields under realistic conditions, considering the influence of high-voltage transmission line conductors. By implementing these numerical techniques in MATLAB, the study aims to assess their accuracy, efficiency, and stability, offering practical solutions for modelling and analysing high-voltage systems. From a numerical solution perspective, the convergence behaviour of the two numerical methods highlights their applicability to different scenarios. As shown in Figure 3, FDM achieves convergence by iteratively updating the grid points based on neighbouring values, with the process terminating when the maximum change in potential falls below a predefined threshold. The FDM plot exhibits a smooth gradient from high potential to low potential. This shows that FDM is efficient in solving steady-state problems. Colour distribution demonstrates a gradient where the highest values (red/yellow) are near the boundaries, indicating a higher electric field near the boundary, and the lowest values (blue) have a low electrical field at the centre. A clear indication of a smooth gradient governed by the elliptic structure of the governing PDE makes the potential function naturally smooth. This smoothness is inherited by the electric field, which is the spatial derivative of the potential. This promises constant, non-abrupt changes in the field intensity throughout space, which is in accordance with the mathematical features of the solution and physical laws.

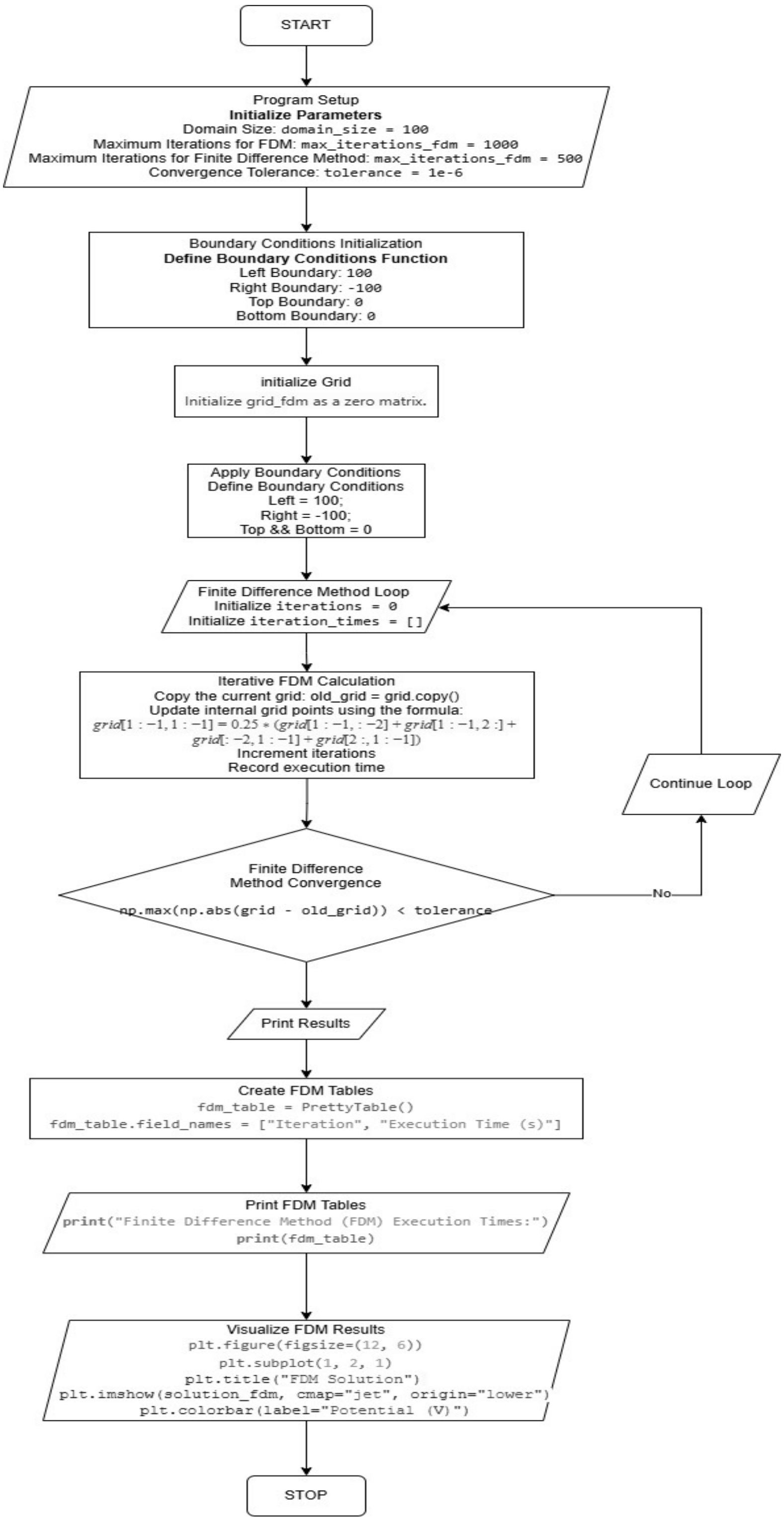

Figure 1. Flowchart of implementation of Laplace equation using Finite Difference Method

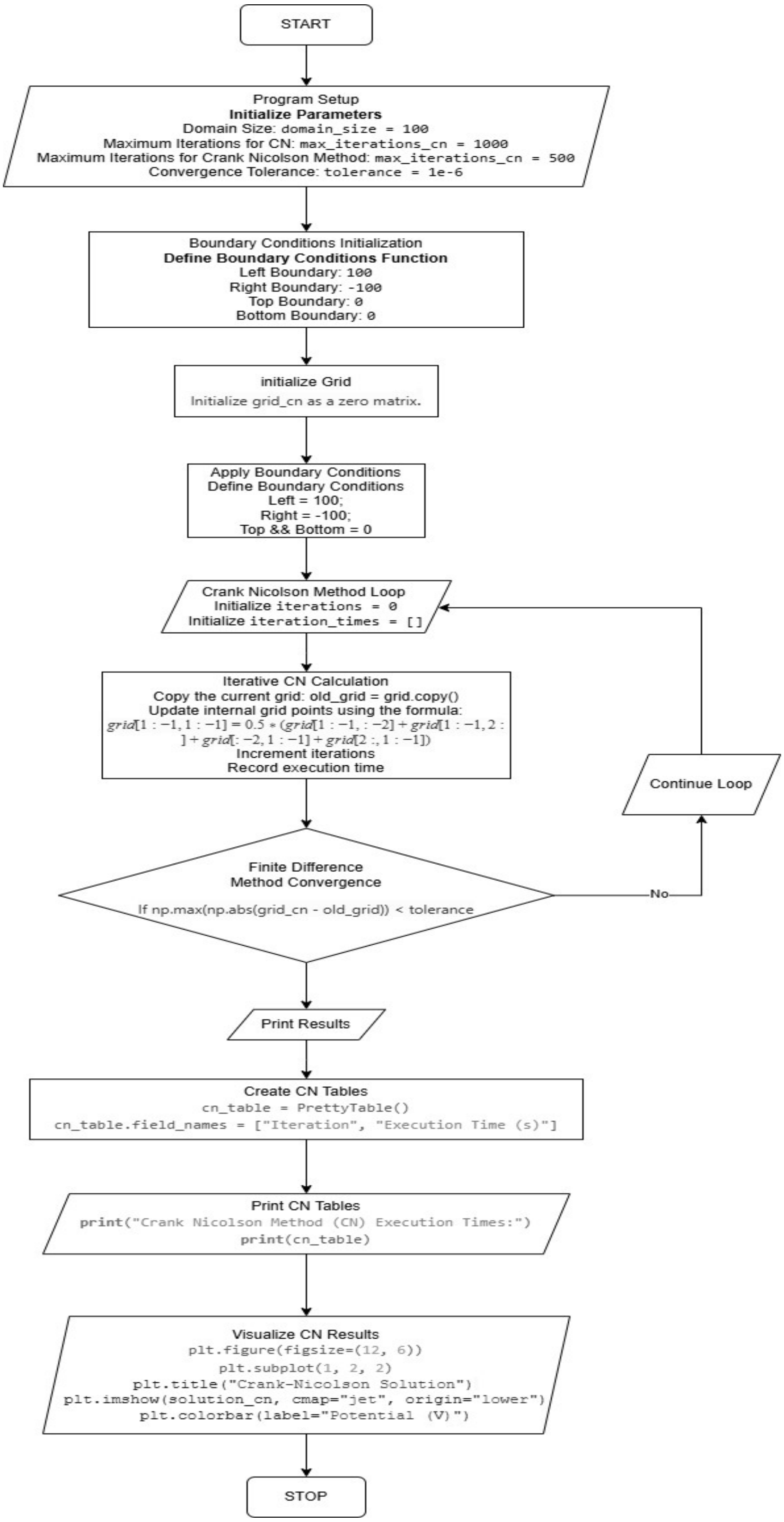

Figure 2. Flowchart of implementation of Laplace equation using Crank-Nicolson Method

On the other hand, the Crank-Nicolson method shown in Figure 4 shows a different potential distribution compared to the FDM; instead of high values on the boundary, the highest values appear at two concentrated regions in the centre while the boundaries remain low. This shows numerical instability, an improper time-step selection, or an issue with boundary conditions. It appears mostly flat and requires solving a linear system at each iteration, which is computationally intensive as it involves solving the tridiagonal matrix; however, the presence of very high values (10^{149}) in the colour bar suggests a divergence issue and computational overflow in the Crank-Nicolson implementation.

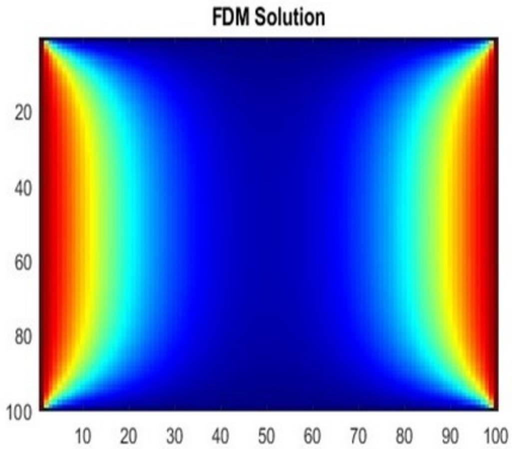

Figure 3. The heatmap shows the solution in Laplace equation using the finite difference method

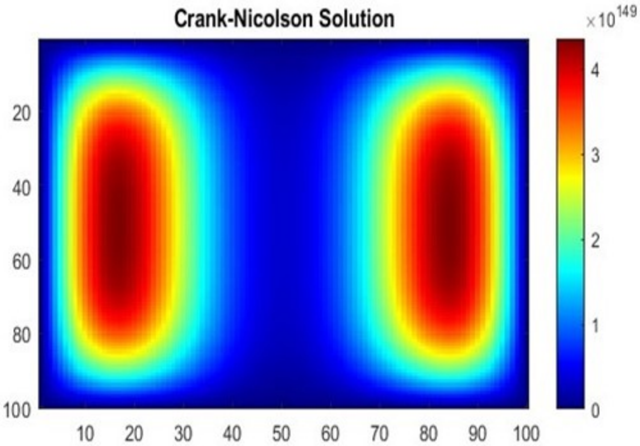

Figure 4. The heatmap shows the solution of Laplace equation using the Crank-Nicolson method

Furthermore, an analysis of the 3D perspective of the same solutions, which makes it easier to visualise the distribution of values, is shown in Figure 5 and Figure 6.

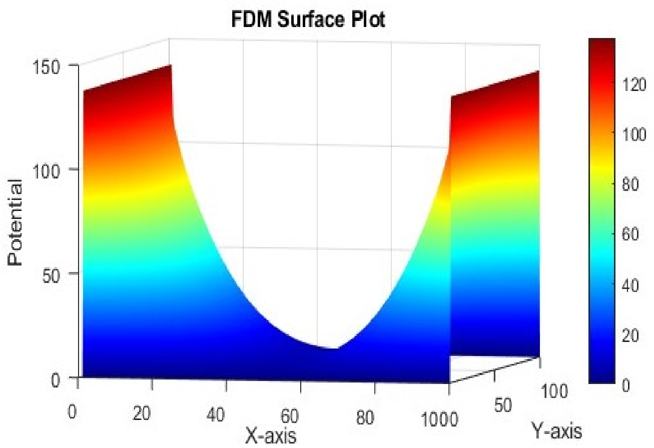

Figure 5: Surface plot shows the solution of Laplace equation using Finite Difference method

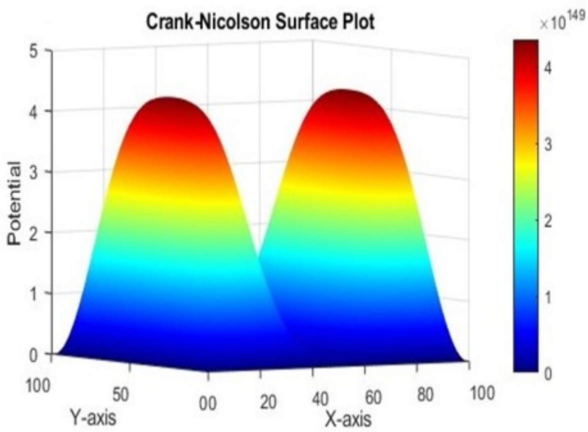

Figure 6: Surface plot shows the solution of Laplace equation using Crank-Nicolson method

Figure 5, which is a surface plot for the FDM, shows a smooth, parabolic-like shape, indicating a stable and well-behaved solution. The highest values (red peaks) are along the edges, while the lowest values (blue valleys) are at the centre. This feature suggests that FDM has properly handled diffusion and converged to a steady-state solution. The Crank-Nicolson plot shown in Figure 6 is

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considerably different from Figure 5. The presence of high peak values suggests a numerical instability problem, as it appears to have exploded to 10^{143}, which is an unphysical and unrealistic solution. This could be linked to matrix inversion instability, which requires solving a complex matrix system.

However, the simulated result of the Laplace equation using FDM aligned well with conductors reflecting high voltages of transmission lines, while the ground and far-field boundaries ensure a realistic simulation of the surrounding environment. The results align well with practical situations, showing significant potential gradients near the conductors and diminishing values at the boundaries. This accurate representation is crucial in understanding the influence of high-voltage lines on nearby structures, ecosystems, and safety guidelines. FDM demonstrates high accuracy in modelling the electrostatic potential, with results closely matching theoretical expectations for sample cases. In terms of efficiency, FDM outperforms Crank-Nicolson for steady-state problems due to its straightforward iterative updates, making it a better choice for modelling steady-state complex problems involving large grids or varying conditions. These differences underscore the importance of selecting the method that best aligns with the specific requirements of the problem. The findings from this study have significant implications for real-world applications. By accurately modelling the electrostatic field distribution, engineers can optimise the design and insulation of high-voltage transmission lines to minimise potential risks and improve efficiency. Despite its strengths, the study has some limitations. The simulations assume idealised boundary conditions and neglect external factors such as weather effects, inductive, resistance or capacitive interactions with neighbouring lines, and material properties of surrounding structures. Additionally, the computational domain size and grid resolution may introduce errors, particularly near the boundaries. Future work could address these limitations by incorporating more realistic conditions and extending the model to capture the effect of external factors. However, this study demonstrates the effective use of FDM and Crank-Nicolson methods for solving and analysing the electromagnetic potential distribution around high-voltage transmission lines. While FDM offers simplicity and efficiency for steady-state solutions, Crank-Nicolson's superior stability is not apparent here due to the nature of the problem and its dynamic scenarios. The comparison of these methods highlights their respective strengths and weaknesses, offering valuable insights for their application in electrical engineering. The results contribute to a better understanding of electrostatic fields in practical settings, with potential applications in safety assessments, design optimisation strategy, and environmental impact analysis.

9. Conclusion

We have successfully analysed and compared the Finite Difference Method (FDM) and the Crank-Nicolson Method for solving the electrostatic potential distribution in the vicinity of high-voltage transmission lines. We focus mainly on the accuracy, convergence rate, computational efficiency, and applicability of these numerical methods to real-world scenarios in Malaysia's power grid infrastructure. FDM showed good accuracy for simpler configurations; its explicit nature might render it ineffective for cases requiring high stability or precision over extended domains. In terms of convergence rate and computational efficiency, the FDM was faster in terms of iterations for straightforward cases without space charge effects, making it suitable for smaller and simpler problems. The time execution comparison indicated that FDM was quicker for small-scale problems, but the superiority of the Crank-Nicolson Method cannot be conclusively established in this context due to the steady-state nature and elliptic structure of the governing equation. Consequently, its applicability remains limited and cannot be fully validated under realistic conditions representative of Malaysia's high-voltage power transmission system. This confirms the applicability of both the Finite Difference Method (FDM) and the Crank-Nicolson Method (CNM) to electrostatic field problems. However, the Crank-Nicolson method is inherently more suited to time-dependent problems involving transient phenomena, where its unconditional stability and second-order accuracy in both time and space offer significant advantages over extended simulations. In contrast, for steady-state electrostatic analysis, the additional computational overhead of the Crank-Nicolson method may not provide substantial benefits compared to more direct methods like FDM.

Conflict of interest

We declare no conflict regarding the publication of the study.

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